Biot and Fourier numbers

In some transient problems, the internal temperature gradients in the body may be quite small and insignificant. Yet the temperature at a given location, or the average temperature of the object, may be changing quite rapidly with time. From eq. (5.1) we can note that such could be the case for large thermal diffusivity α.

A more meaningful approach is to consider the general problem of transient cooling of an object, such as the hollow cylinder shown in figure 5.1.

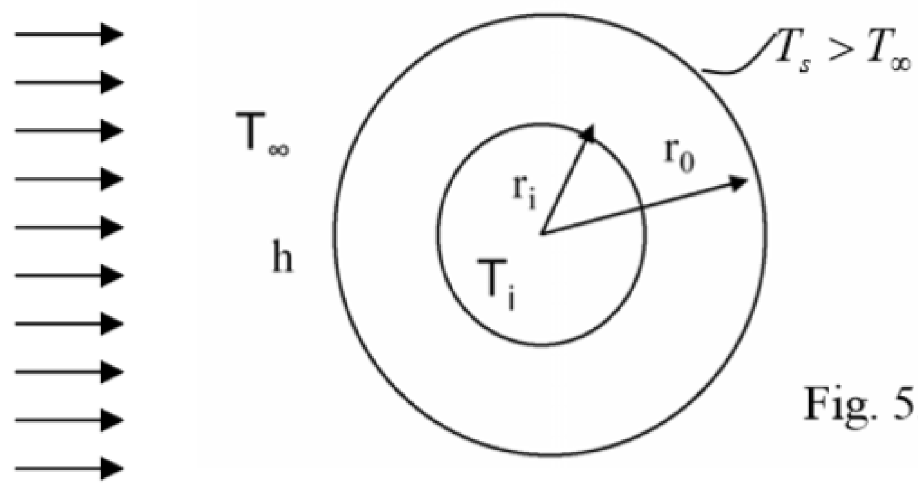


Fig. 5.1

For very large r_i, the heat transfer rate by conduction through the cylinder wall is approximately

$$q \approx -k(2\pi r_o l)\left(\frac{T_s - T_i}{r_o - r_i}\right) = k(2\pi r_o l)\left(\frac{T_i - T_s}{L}\right) \tag{5.3}$$

where l is the length of the cylinder and L is the material thickness. The rate of heat transfer away from the outer surface by convection is

$$q = \bar{h}(2\pi r_o l)(T_s - T_\infty) \tag{5.4}$$

where $\bar{h}$ is the average heat transfer coefficient for convection from the entire surface. Equating (5.3) and (5.4) gives

$$\frac{T_i - T_s}{T_s - T_\infty} = \frac{\bar{h}L}{k} = \text{Biot number} \tag{5.5}$$

The Biot number is dimensionless, and it can be thought of as the ratio

$$\mathbf{Bi} = \frac{\textbf{resistance to internal heat flow}}{\textbf{resistance to external heat flow}}$$

Whenever the Biot number is small, the internal temperature gradients are also small and a transient problem can be treated by the "lumped thermal capacity" approach. The lumped capacity assumption implies that the object for analysis is considered to have a single mass-averaged temperature.

In the derivation shown above, the significant object dimension was the conduction path length, $L = r_o - r_i$. In general, a characteristic length scale may be obtained by dividing the volume of the solid by its surface area:

$$L = \frac{V}{A_s} \qquad (5.6)$$

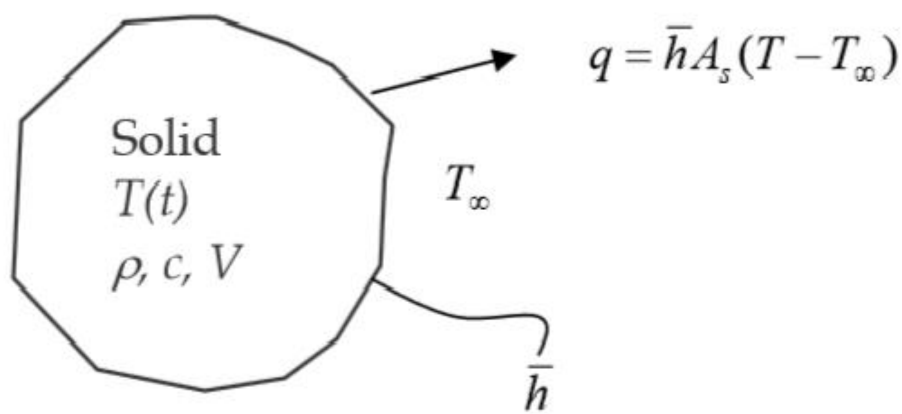
Using this method to determine the characteristic length scale, the corresponding Biot number may be evaluated for objects of any shape, for example a plate, a cylinder, or a sphere. As a thumb rule, if the Biot number turns out to be less than 0.1, lumped capacity assumption is applied.

In this context, *a dimensionless time*, known as the **Fourier number**, can be obtained by multiplying the dimensional time by the thermal diffusivity and dividing by the square of the characteristic length:

$$\textbf{dimensionless time} = \frac{\alpha t}{L^2} = \textbf{Fo} \qquad (5.7)$$

Lumped thermal capacity analysis

The simplest situation in an unsteady heat transfer process is to use the lumped capacity assumption, wherein we neglect the temperature distribution inside the solid and only deal with the heat transfer between the solid and the ambient fluids. In other words, we are assuming that the temperature inside the solid is constant and is equal to the surface temperature.



The solid object shown in figure is a metal piece which is being cooled in air after hot forming.

Now, if Biot number is small and temperature of the object can be considered to be uniform, this equation can be written as

$$\bar{h}A_s\left[T(t)-T_\infty\right]dt=-\rho cVdT \quad (5.8)$$

or,

$$\frac{dT}{(T-T_\infty)}=-\frac{\bar{h}A_s}{\rho cV}dt \quad (5.9)$$

Integrating and applying the initial condition $T(0)=T_i$,

$$\ln\frac{T(t)-T_\infty}{T_i-T_\infty}=-\frac{\bar{h}A_s}{\rho cV}t \quad (5.10)$$

Taking the exponents of both sides and rearranging,

$$\frac{T(t)-T_\infty}{T_i-T_\infty}=e^{-bt} \quad (5.11)$$

where

$$b=\frac{\bar{h}A_s}{\rho cV} \qquad (1/\text{s}) \quad (5.12)$$